2.d. Time-weighted and Flow-weighted Mean Concentrations

Introduction

"What is the average concentration of nitrate in the river?" That is a frequently asked question, and river data sets such as these can certainly provide an answer.   The concentration exceedency curves and associated data sets do provide information on percentiles of concentration, including the median (50% time exceedency) concentration. However, they do not provide average concentrations.

Calculation of average concentrations would be easy if each sample had equal weight in determining the average. However, with the stratified sampling program used in loading studies, each sample does not have equal weight. Some samples may represent one or more days, while others represent only a few hours. These different "weights" must be taken into account as averages are calculated.

In river systems, two types of "average" or "mean" concentrations can be considered. One is the average concentration as seen by fish in the stream or by public water supplies that draw upon the river for their raw water. A second kind of average would be from the perspective of the water body that receives the discharge from the stream. For example, what is the average concentration of water discharged from the Sandusky River into Sandusky Bay?

The first kind of average requires calculation of a time-weighted mean concentration (TWMC) while the second requires calculation of a flow-weighted mean concentration (FWMC).

Time-weighted Mean Concentrations

Calculation of TWMC requires information on the concentration in each sample plus the sample time window for each sample. The equation for calculating the TWMC is

$$TWMC = \frac{\sum_{1}^{n}(c_i * t_i)}{\sum_{1}^{n}(t_i)}$$

where c_i = concentration is the ith sample

t_i = time window for the ith sample

Through the use of the above equation, the concentration in each sample is weighted by the period of time it represents.

Flow-Weighted Mean Concentrations

For the calculation of the FWMC, data on the concentration, sample time window and flow are required for each sample. The equation for calculating the FWMC is

$$FWMC = \frac{\sum_{1}^{n}(c_i * t_i * q_i)}{\sum_{1}^{n}(t_i * q_i)}$$

where q_i = flow in the ith sample

With this equation the concentration in each sample is weighted by both the time and the flow that accompanied it. The FWMC represents the total load for the time period divided by the total discharge for the time period.

The ratio of FWMC to TWMC indicates whether a pollutant tends to increase in concentration as flow increases. If the FWMC>TWMC, that pollutant, on average, increases with increasing flow.

An Example of Time-Weighted and Flow -Weighted Mean Concentration Calculations

The adjacent table shows an Excel worksheet used to calculate TWMC and FWMC values. The data are taken from the Sandusky River and cover the same runoff event shown in Hydrographs, Sedigraphs and Chemographs.

In this example the FWMC (435.0 mg/L) is much higher than the TWMC (157.9 mg/L) . It is evident from the data that the samples with the highest flow also had the highest concentrations.

This example also shows the differences between the TWMC (157.9 mg/L) and the average concentration (240.4 mg/L) that is calculated if the 31 samples are given equal weight. This illustrates the errors introduced by not recognizing that these samples are produced by a stratified sampling program. Those errors are avoided by calculating a TWMC.

As noted on the previous page, the sums of Column F (Flow*Time) and Column G (Flow*Time*SS Conc.) represent the total flow volume and the total suspended solids load discharged during the time interval. Those values need only be multiplied by a conversion factor to obtain appropriate units of volume (e.g. cubic meters) or loads (e.g. kilograms).

Note: The Report Summary program of AnalysisTemplatev3 includes calculation of the TWMC and the FWMC for the stream, the parameter, and the time interval selected.

	A		B	D	E	F	G
1	Datetime	Time Window	Flow	SS	Time*Conc	Flow*Time	Flow*Time*Conc
2	6/2/1993 20:00	1.00	313	49.0	49.00	313.00	15337.0
3	6/3/1993 20:00	1.00	279	33.4	33.40	279.00	9318.6
	Rows 4-10 deleted						
11	6/8/1993 20:00	0.33	534	246.0	82.00	178.00	43788.0
12	6/9/1993 4:00	0.33	1980	1160.0	386.67	660.00	765600.0
13	6/9/1993 12:00	0.33	4940	1170.0	390.00	1646.67	1926600.0
14	6/9/1993 20:00	0.33	8270	852.0	284.00	2756.67	2348680.0
15	6/10/1993 4:00	0.33	8020	641.0	213.67	2673.33	1713606.7
16	6/10/1993 12:00	0.33	6750	389.0	129.67	2250.00	875250.0
17	6/10/1993 20:00	0.33	4970	319.0	106.33	1656.67	528476.7
18	6/11/1993 4:00	0.33	3230	315.0	105.00	1076.67	339150.0
19	6/11/1993 12:00	0.33	2620	268.0	89.33	873.33	234053.3
20	6/11/1993 20:00	0.67	2190	221.0	147.33	1460.00	322660.0
	Rows 21-28 deleted						
29	6/19/1993 20:00	1.00	313	42.1	42.10	313.00	13177.3
30	6/20/1993 20:00	0.83	262	43.6	36.33	218.33	9519.3
31	6/21/1993 12:00	0.50	389	84.5	42.25	194.50	16435.3
32	6/21/1993 20:00	0.50	509	186.0	93.00	254.50	47337.0
33							
34	Column sum	19.83	52523.00	7453.10	3132.03	22291.67	9697143.23
35							
36	TWMC = 31313.03/19.83 = 157.9 mg/L						
37	FWMC = 9697143.23/22291.67 = 435.0 mg/L						
38	Unweighted average concentration for 31 samples = 7453.10/31 = 240.4 mg/L						